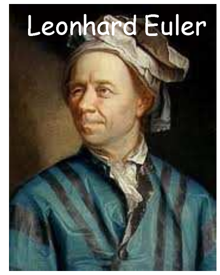
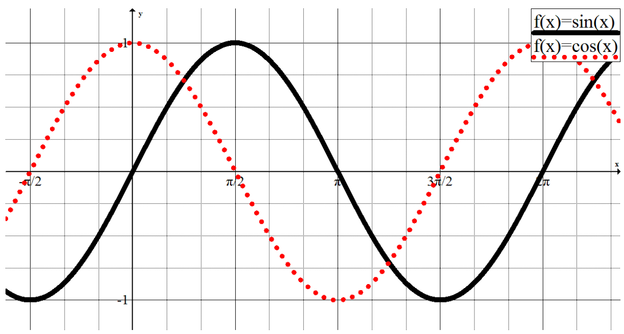
1.8 e^x & lnx



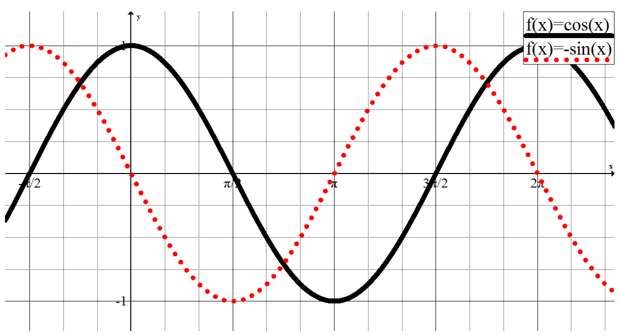
Summary from Investigating Trig and Exp Functions:

link to 1.7 investigation answers

IF f(x) = sinx then f'(x)= cosx



IF f(x) = cosx then f'(x)= - sinx



Effect of Transformations on Base Trig Functions:

vertical translation - no effect on derivative

horizontal translation- derivative has same horizontal translation

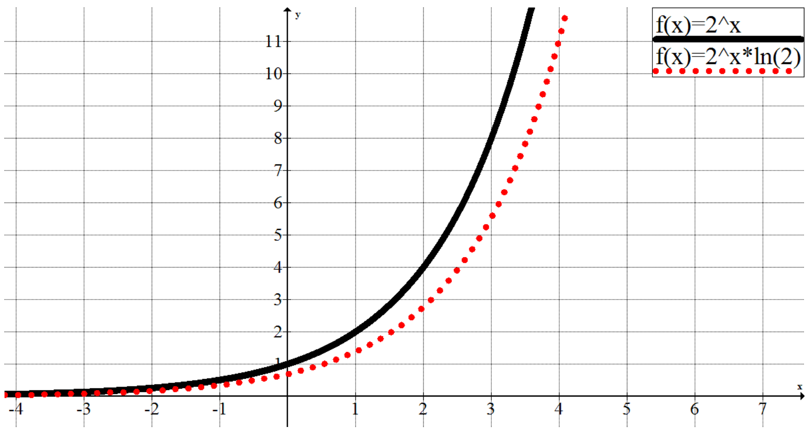
vertical stretch/reflection - derivative has same vertical stretch/reflection

**horizontal stretch/reflection
-derivative has the same transformations BUT ALSO impacts the vertical stretch of the derivative function
-a shorter period results in steeper tangents, a longer period results in flatter tangents...this affects the amplitude of the derivative function

Feb 16-9:51 AM

IF $f(x) = 2^x$, then $f'(x) = 2^x (k)$, where $k = \ln 2$.
This represents a vertical compression of f(x).

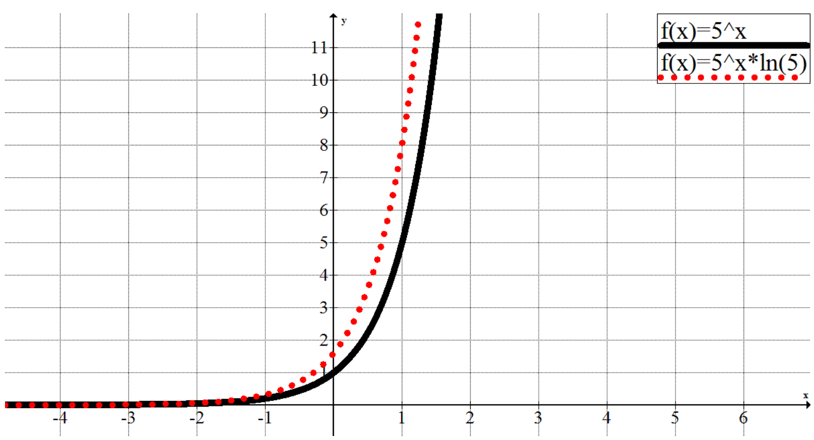
$$k = \frac{f'(x)}{f(x)} = \ln 2 \doteq 0.69$$



$\therefore k < 1$, compression

IF $f(x) = 5^x$, then $f'(x) = 5^x (k)$, where $k = \ln 5$.
This represents a vertical stretch of f(x).

$$k = \frac{f'(x)}{f(x)} = \ln 5 \doteq 1.61$$



$k > 1$
$\therefore$ stretch

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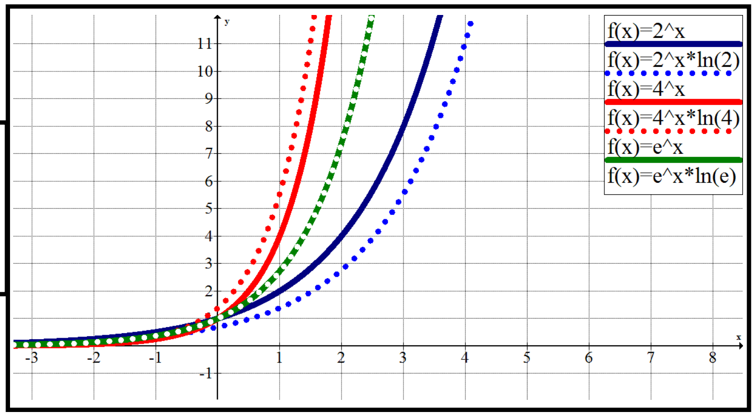
Q: If $f(x) = a^x$, is there a value of "a" where the derivative function would result in the same curve as the original function?
(ie. no stretch or compression)

Graph of $f(x)=e^x$



When "a" is approximately equal to 2.72 or "e", the derivative function is equivalent to the original function.

When $f(x)=2.72^x$, the derivative function $f'(x)=2.72^x$ (same as f(x))
When the base is close to 2.72 the value of k approaches 1
...therefore, there is no compression or stretch.

Called Euler's Number (thus the "e").

This is one of many definitions of the number "e".

e = the base of an exponential function whose derivative function is itself

e = 2.71828182845904523536028747135266249775724709369995957496...

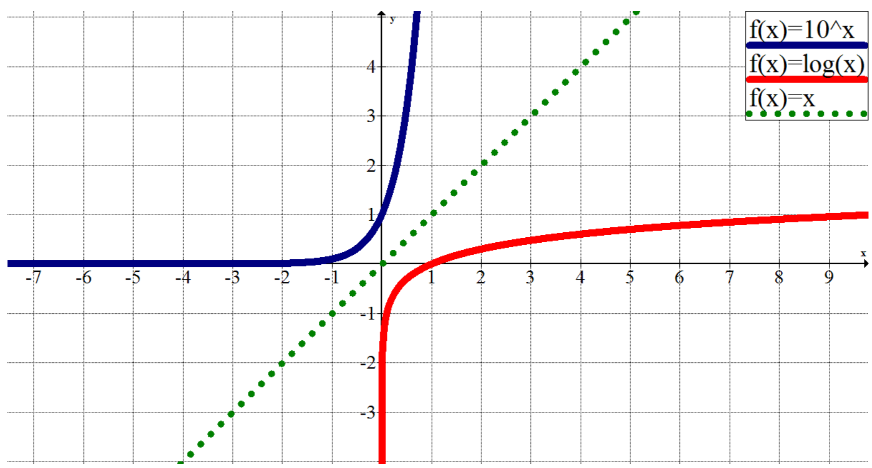
e is an irrational number (it cannot be written as a fraction, it never ends, there is no repeating pattern)

Recall: The inverse of $y=a^x$ can be written:

$x = a^y$ (exponential form)

$y = \log_a x$ (logarithmic form)

Graphically: $y = a^x$ and $y = \log_a x$ are reflections in the line $y=x$.



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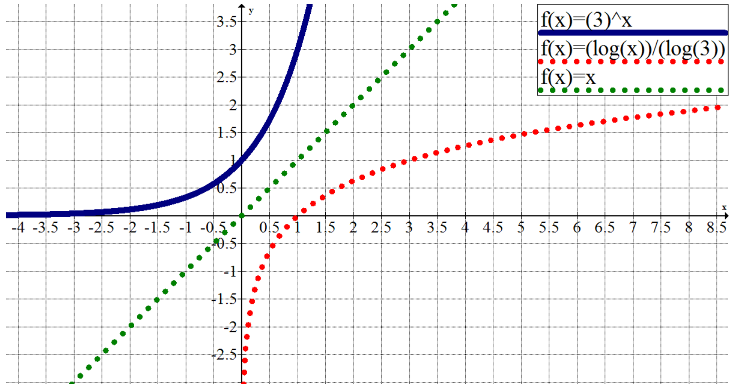
Ex. 1 Determine the inverse of each function.
Graph f(x) and f'(x).

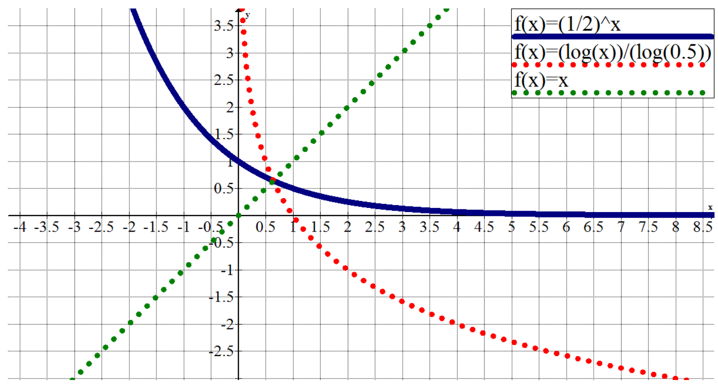
a) $f(x) = 3^x$

$f^{-1}(x) = \log_3 x$



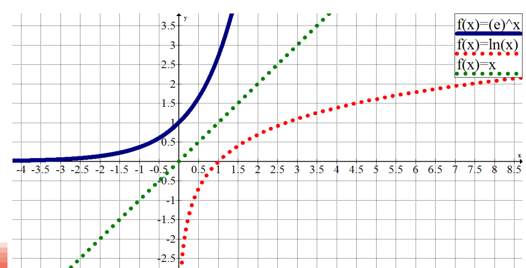
b) $f(x) = \left(\frac{1}{2}\right)^x$

$f^{-1}(x) = \log_{\frac{1}{2}} x$



c) $f(x) = e^x$

$f^{-1}(x) = \log_e x$
$= \ln x$



ln is also known as the natural log

Log Laws:

$\log_a m + \log_a n = \log_a mn$

$\log_a\left(\frac{m}{n}\right) = \log_a m - \log_a n$

$\log_a m^p = p(\log_a m)$

$\log_a 1 = 0$

$\log_a a^x = x$

$a^{\log_a x} = x$

Change of Base Formula:

recall: $\log_b a = \frac{\log_m a}{\log_m b}$

when m = e,

$\log_b a = \frac{\log_e a}{\log_e b}$

$= \frac{\ln a}{\ln b}$

Ex. 2 Simplify/evaluate each of the following.

a) $\log_5 1$ $=0$

b) $\log_6 6^x$ $=x$

c) $6^{\log_6 x}$ $=x$

d) $\ln e$ $=1$

e) $\ln 1$ $=0$

f) $e^{\ln x}$ $=x$

g) $\ln e^x$ $=x$

$\log_e e^x$

Ex. 3 Simplify and evaluate each of the following.

a) $\log_6 2 + \log_6 3$
$=\log_6 6$
$=1$

b) $\log_2 24 - \log_2\left(\frac{3}{4}\right)$
$=\log_2 32$
$=5$

$24 \cdot \frac{4}{3}$
$=32$

c) $2\log_2\sqrt{8} - 2\log_2 4$
$=2\log_2 2^{\frac{3}{2}} - 2\log_2 2^2$
$=2\left(\frac{3}{2}\right) - 2(2)$
$=-1$

d) $3\ln 2 - 3\ln 5$
$=3(\ln 2 - \ln 5)$
$=3\ln\left(\frac{2}{5}\right)$
$=\ln\frac{8}{125}$

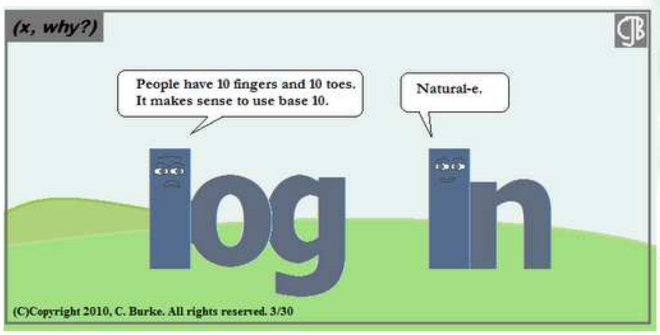
Ex. 4 Use your calculator to evaluate.

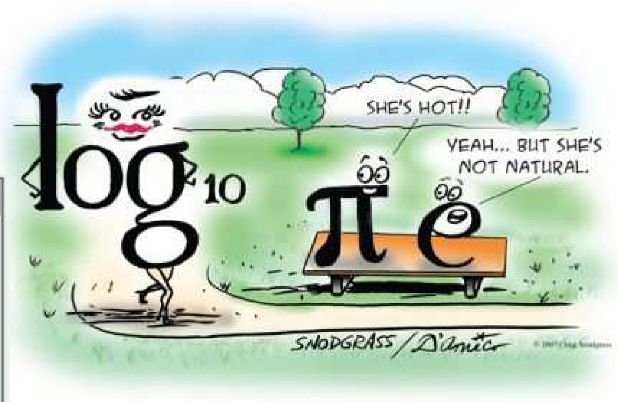
a) $\log_2 18$
$=\frac{\log 18}{\log 2}$
$\doteq 4.17$

b) $\log_5 3$
$\doteq 0.68$

c) $\log_e 10$
$=\frac{\log 10}{\log e}$
OR
$=\ln 10$
$\doteq 2.3$

Homework: Handout





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Feb 16-10:00 AM